Geodezijske na kuli (19.III.15)

$$\int \sqrt{1+(\varphi'(\theta))^2 \sin^2\theta}\, d\theta \qquad F=\sqrt{1+\varphi'^2\sin^2\theta}, \quad \frac{\partial F}{\partial\varphi}-\frac{d}{d\theta}\frac{\partial F}{\partial\varphi'}=0$$

$$\frac{d}{d\theta}\frac{\partial F}{\partial \varphi'}=\frac{d}{d\theta}\frac{2\varphi'\sin^2\theta}{2\sqrt{1+\varphi'^2\sin^2\theta}}=\frac{(\varphi''\sin^2\theta+\varphi' 2\sin\theta\cos\theta)\,\text{nazivnik}}{\text{nazivnik}}$$

$$-\varphi'\sin^2\theta\left(\frac{2\varphi'\varphi''\sin^2\theta+\varphi'^2\sin 2\theta}{2\sqrt{\ }}\right)=0$$

množenje

$$\left(\varphi''\sin^2\theta+\tfrac{1}{2}\varphi'\sin 2\theta\right)(1+\varphi'^2\sin^2\theta)-\varphi'^2\sin^4\theta\,\varphi''-\tfrac{1}{2}\varphi'^3\sin 2\theta\sin^2\theta=0$$

$$\varphi''\sin^2\theta+\cancel{\varphi''\varphi'^2\sin^4\theta}+\tfrac{1}{2}\varphi'\sin 2\theta+\cancel{\tfrac{1}{2}\varphi'^3\sin 2\theta\sin^2\theta}-$$

$$-\cancel{\varphi'^2\sin^4\theta\,\varphi''}-\cancel{\tfrac{1}{2}\varphi'^3\sin 2\theta\sin^2\theta}=0$$

$$\varphi''\sin^2\theta+\tfrac{1}{2}\varphi'\sin 2\theta=0, \quad \varphi''\sin^2\theta+\varphi'\sin\theta\cos\theta=0$$

$$\varphi''=-\varphi'\,\mathrm{ctg}\,\theta, \quad \varphi'=g, \quad \frac{dg}{g}=-\mathrm{ctg}\,\theta\, d\theta, \quad \ln g=-\int \mathrm{ctg}\,\theta\, d\theta$$

$$g=e^{-\int \mathrm{ctg}\,\theta\, d\theta}, \quad \varphi=\int e^{-\int \mathrm{ctg}\,\theta\, d\theta}\, d\theta \qquad \frac{dg}{d\theta}=\frac{dg}{du}\frac{du}{d\theta}=\frac{dg}{du}\frac{-1}{1+\theta^2}$$

$$u=\mathrm{ctg}\,\theta, \quad \theta=\mathrm{arcctg}\, u$$

$$\frac{-g'}{1+\theta^2}=gu \qquad \frac{\varphi'\sin^2\theta}{\sqrt{1+\varphi'^2\sin^2\theta}}=C, \quad \varphi'^2\sin^4\theta=C+C\varphi'^2\sin^2\theta$$

$$\varphi'^2\sin^2\theta(\sin^2\theta-C)=C$$

$$\varphi'=\frac{\sqrt{C}}{\sin\theta\sqrt{(\sin^2\theta-C)}}=\frac{c}{\sin\theta\sqrt{(\sin^2\theta-c^2)}}, \quad u=\frac{\cos(\theta)}{\sin\theta}, \quad du=\frac{-1}{\sin^2\theta}d\theta$$

$$\frac{d\varphi}{du}=-\frac{\sin\theta\, c}{\sqrt{(\sin^2\theta-c^2)}}, \quad \varphi=\int\frac{-c\,du}{\sqrt{1-c^2/\sin^2\theta}}= \qquad d\theta=-\sin^2\theta\, du, \quad \frac{1}{\sin^2\theta}=1+\mathrm{ctg}^2\theta$$

$$=\int\frac{-c\,du}{\sqrt{1-c^2(1+u^2)}}=-\int\frac{du}{\sqrt{a^2-u^2}}\quad\left(a=\frac{\sqrt{1-c^2}}{c}\right)$$

$$=-\arccos\left(\frac{u}{a}\right)+\varphi_0$$

→ $\boxed{\operatorname{ctg}\theta = a\cos(\varphi-\varphi_0)}$, $\frac{x}{y} = \operatorname{ctg}\theta$

$\cos\varphi = \frac{x}{r\sin\theta} = \frac{x}{r\sqrt{1-\frac{z^2}{r^2}}} = \frac{x}{\sqrt{r^2-z^2}}$

$\frac{x}{y} = a\frac{x}{\sqrt{r^2-z^2}}$ $\quad x\sqrt{r} - \frac{xz^2}{r} = xyaz$

$r^2 -$ $\quad \frac{x^2}{y^2} = \frac{a^2x^2}{r^2-z^2}$ $\quad x^2 r - x^2z^2 = r^2a^2x^2y^2$

$r^2 - z^2 = r^2a^2y^2$

$\cos\varphi = \frac{x}{r\sin\theta}$

$\cos\theta = \frac{z}{r}$, $\quad \sin\theta = \frac{y}{r\sin\varphi} = \frac{x}{r\cos\varphi}$, $\quad \operatorname{ctg}\theta = \frac{z}{r\sin\theta}$ // $\frac{z}{x}$

$\frac{z \cdot r\sin\theta}{ry} = \frac{x\sin\varphi}{r} = x$

$\cos\theta = \frac{x}{r} = \frac{z}{r}$

$\operatorname{ctg}\theta = a\cos(\varphi-\varphi_0) = a\cos\varphi\cos\varphi_0 + a\sin\varphi\sin\varphi_0 =$

$= A\cos\varphi + B\sin\varphi$, $\quad A = a\cos\varphi_0$, $B = a\sin\varphi_0$

mnożymy obie strony przez $R\sin\theta$:

$z = Ax + By$ // jednocześnie te punkty leżą na $x^2+y^2+z^2 = R^2$